OPTIMUM FISCAL AND MONETARY POLICY USING THE MONETARY OVERLAPPING GENERATION MODELS

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ABSTRACT
Government financing is one of the most important issues that any economy is faced with it. In a capitalism system, the government can only finance their expenditure from taxes. Taxes could levy on any goods, services and activity but the important problem is the good and how much one of them must pay the tax? According to Characteristics of each good, the effect of levying the tax on it in economy is different. In this article we show the money is the best goods that government can levy the tax on it. If the government want collect the constant quantity of tax, welfare loose of levying the tax on money is smaller than other goods. These results achieve from Frank Ramsey paper and John Maynard Keynes book. Other problem than answered this article is determining the optimum rate of tax on money (inflation tax) in an overlapping generation model that characterized the four sector economy.
Keywords: Inflation Tax, OLG Model, Fiscal policy, Monetary Policy

INTRODUCTION
Governments use fiscal policy tools to achieve their goals (both long run and short run goals). The main goal of the government has been maximize welfare of society during the time. To achieve this goal, governments must make different policies in different areas. In order to finance the expenses of policies, governments need to mobilize financial resources. Tax is the only permissible source of funds to finance the government expanding in capitalist economy. This is so important issue that governments determine which goods are taxable and how much tax they should get for goods. In order to answer these questions we determine features and type of taxable goods by using existing literature at the first and then Optimum level of taxes will be determined by using Overlapping generations model.

Which goods are taxable?
When the government levies a tax on a good, it will affect on Optimization decisions of consumers and Leads to loss of social welfare because of financial recourses transfer from taxpayers (households) to government. In as much as goods have the unique characteristics, therefore levied tax on a good or a group of the goods leave different welfare effects. So government should levied tax on goods that has minimum welfare loss on society. To identify these Features three main sources are used.
Frank Ramsey article (A contribution to the theory of taxation, 1926).
Keynes’s General Theory
David Ricardo’s Principles of Political Economy and Taxation
At the first, we use Frank Ramsey article. He found an equation based on Optimization consumer behavior with regard to the taxation on goods in the economy in his paper. This equation is determinative the characteristics of goods that if the tax levied on them, they will imposed smallest deadweight loss to society. Mentioned equation is:

\[
\theta = \frac{\mu_1}{\theta \left( \frac{1}{\varepsilon_1} + \frac{1}{\rho_1} \right)}
\]

Where, \( \varepsilon_1 \) is supply elasticity of goods, \( \rho_1 \) is demand elasticity of good, \( \mu_2 \) is tax rate levied on goods and \( \theta \) is constant.
Based on this equation, first characteristics of the goods were determined. Another important required characteristic is using of this goods or a group of goods should be enough to enable government to finance the required financial resources through taxation on them. Another required Feature of these goods is Low elasticity of substitution. This feature is required due to, providing taxation on these goods, other goods don’t replace them and does not cause to unexpected changes in Sources of government tax.

John Maynard Keynes has expressed some important points about properties of money as one of the goods in economy in his book (The General Theory of Employment, Interest and Money).
Production elasticity of money is zero
Compensated elasticity of money is zero
The government can be obtained a huge and certain amount of recourses by printing money. This resource of earning money for government called seignorage. This point is that government can earn profit by this process because expense of publishing new money is less than nominal value of that the profit arising from publishing new money called seigniorage and in fact is one of the financial resources of government in developing countries.
The surplus value of money than its production cost is called seignorage. The perfect money like gold, has equal value metal that used and as a result mint of this metal provide less seignorage but paper money and alternative coins provide more seigniorage. Thus seigniorage can be the part of governmental income that government acquired from decline of real value of money by inflation and decrease of governmental domestic debts. What the government gets from printing money is not inflation tax but is seigniorage, but yet these two concept are closely related, so that when inflation and money growth rate considered equal, this two concepts become same and inflation tax is equal to seigniorage. Inflation tax reduces the cash value by publishing money. Missing value of money is exactly the same of published money by government. Inflation rate is less than seigniorage. The difference between printing money and inflation rate is called inflation tax. This extra printing money is same as tax and reduces the value of the money published in the previous periods. This reduction acts like tax each person has more money pay more taxes. The government cans uses of this kind of tax for financial expenses. In this regard, designed the economic that government done financial expenses by publishing money. Because this commodity has a zero production and substitution elasticity then this good (money) can be consider as a Ramsey’s prepare good for taxation. According to Frank Ramsey model, taxation from this commodity prepares minimal reduction and welfare losses. In next section an OLG
model will be constructing for an economy which in that government financing there budget only with inflation tax.

3. THE MODEL
In this section we set up the economy version of the overlapping generation’s model that the government finances their policies expenditure only with use of seigniorage. This economy construct on any other assumes.

1- This economy have 4 sectors (Government, household, production and monetary).
2- There is no population growth.
3- Government allocate equally the money among the elder
4- Agents are two period-lived
5- Time is discrete
6- Agents supply inelastically one unit of labor in their youth and are retired during old age

Part 1: Government
At each period, the government published the money to finance their policy expenditure. The budget constraint of the government is as follows:

\[ P_t G_t = M_t - M_{t-1} = \lambda M_{t-1} \]

Let \( w_t \) denote the real price of labor, of the consumption during youth. Agents can invest their savings in two assets: a nominal asset (money \( M_t \)) and a real asset (physical capital) \( S_t \) which will only be productive at date \( t+1 \), the budget constraint of a young agent at date \( t \) writes in real terms:

\[ C_t + S_t + \frac{M_t}{P_t} = w_t \]

When old, agents simply consume their real wealth:

\[ d_{t+1} = R_{t+1} S_{t+1} + \frac{M_{t+1}}{P_{t+1}} + \frac{G_{t+1}}{P_{t+1}} \]

where \( dt+1 \) is the consumption of produced good at date \( t+1 \) by an agent born at date \( t \) and \( Rt+1 \) is the real gross return on investment in capital.

CIA Constrain:
The usual CIA constraint implies that at each date \( t \) agents must hold money balances in order to finance consumption at this date. In a model where life-span is two periods, such a condition forbids holdings of non-monetary assets. Following Hahn and Solow (1995), we assume that a fraction equal to \( \mu \) that \((1 > \mu > 0)\) of consumption during old age is financed by money balances held at the beginning of old age.

\[ M_t = \mu P_{t+1} d_{t+1} \]

Production function
The production function is Cobb-Douglas:
\[ Y_t = F(K,L_t) = AK_t^a L_t^{1-a} \]  

(6)

With \(0 < a < 1\), \(Y\) is output, \(K\) is capital, \(L\) is labor and \(\alpha\) is production elasticity of capital. In the production sector, labor and capital markets are in equilibrium and the marginal productivity of labor and capital equal to the wage and the return on capital respectively. So we have:

\[ w_t = F'_L(k_t, l_t) = A(1-a)(k_t)^a \]  

(7)

\[ R_t = F'_K(k_t, l_t) = A\alpha(k_t)^{\alpha-1} \]  

(8)

1. Market Money:

   The equilibrium relationship is:

   \[ NM_t = \bar{M}_t = (1+\lambda_t)M_{t-1} \]  

   (9)

   So:

   \[ C_t = (1-a)w_t \]  

   (10)

   \[ S_t = aw_t - m_t \]  

   (11)

   \[ m_t = \mu \frac{P_{t+1}}{P_t} d_{t+1} \]  

   (12)

   \[ d_{t+1} = \frac{1}{(1-\mu(1+\lambda))} R_{t+1} S_t \]  

   (13)

   In Equation 10, \((1-a)\) is percent of wages that an individual consumes at this time. Substitute equation (13) in equation (5), we have:

   \[ \frac{1}{NP_t} \bar{M}_{t-1} = \bar{M}_t \]  

   (14)

   In this equation:

   \[ B = \frac{\mu}{1-\rho} A \alpha \]  

   (15)

   and

   \[ \rho = \mu(1+\lambda) \]  

   (16)

   so:

   \[ m_t = \frac{M_t}{P_t} = (1+\lambda_t)Bk_t^\alpha \]  

   (17)

   Using (11, 17 eq.), we have:

   \[ k_{t+1} = S_t = ZB(\lambda_t - \lambda_t\lambda_t)k_t^\alpha \]  

   (18)
That
\[ Z = \frac{a(1-\omega)+\alpha}{\alpha} \]  \hspace{1cm} (19)

And
\[ \lambda = \frac{a(1-\omega)(1-\mu)-\mu\lambda}{(1-\omega)\mu+\mu\lambda} \] \hspace{1cm} (20)

Using this equation government spending is:
\[ G_t = \lambda N \frac{R_t}{p_t} = \lambda N B k_t^\alpha \] \hspace{1cm} (21)

After reaching equilibrium relations; we want to determine the optimal values of the variables. The purpose is to maximize consumer welfare. Therefore, we can write the social welfare function at the present time as follows:
\[ N \sum_{t=0}^{\infty} \delta^t [(\ln c_t) + (\ln d_t + b \ln G_t)] \] \hspace{1cm} (22)

The parameter \( b \) is an indicator of the individuals’ welfare for the public good. Equilibrium relationship in the welfare function is as follows:
\[ c_t = (1-a)w_t = (1-a)(1-\alpha)Ak_t^\alpha \]  \hspace{1cm} (23)
\[ d_t = \frac{1}{1-\rho} R_t S_{t-1} = \frac{aA}{1-\rho} k_t^\alpha \] \hspace{1cm} (24)
\[ G_t = BN \lambda_t k_t^\alpha \] \hspace{1cm} (25)

Since \( k_t \) is the common factor in this entire equilibrium relation. Therefore we can take out common factor from each equation. On the other; the social welfare function is the logarithmic form, we can take the logarithm of the equilibrium relations, and converted to usable form the social welfare function.

The equilibrium log and we will;
\[ \ln c_t = \alpha \ln k_t + C_1 \] \hspace{1cm} (26)
\[ \ln d_t = \alpha \ln k_t + \ln B + C_2 \] \hspace{1cm} (27)
\[ \ln G_t = \ln \lambda_t + \alpha \ln k_t + \ln B + C_3 \] \hspace{1cm} (28)
This enables us to write the social welfare function (up to a constant term) as:

\[ N \sum_{t=0}^{\infty} \delta^t [\alpha(2 + b_1) \ln k_t + b \ln \lambda_t] \quad (29) \]

And the equation of motion for capital as:

\[ k_{t+1} = ZB(\bar{\lambda} - \lambda_t)k_t^\alpha \quad (30) \]

Therefore we have:

\[ \ln k_{t+1} = \alpha \ln k_t + \ln(\bar{\lambda} - \lambda_t) + \ln B \quad (31) \]

Furthermore, in order to avoid an irrelevant solution, we’ll also add a positive constraint as:

\[ \ln k_t > 0 \]

We now maximize \( N \sum_{t=0}^{\infty} \delta^t [\alpha(2 + b_1) \ln k_t + b \ln \lambda_t] \) with respect to \( \lambda_t \) so that \( \ln k_{t+1} = \alpha \ln k_t + \ln(\bar{\lambda} - \lambda_t) + \ln B \) holds at each date \( t \).

The Lagrangian at date \( t \) writes:

\[ L_t = \alpha(2 + b) \ln k_t + b \ln \lambda_t - (1 + b) \ln (1 - \alpha) + \delta q_{t+1} (\alpha \ln k_t + \ln(\bar{\lambda} - \lambda_t) - \ln(1 - \alpha)) - q_t \ln k_t \]

\[ (32) \]

The first order conditions for the Lagrangian at date \( t \) writes:

\[ \frac{\partial L_t}{\partial \lambda_t} = \frac{b}{\lambda_t} + \frac{\mu + b \mu}{(1 - \mu - \lambda \mu)} - \frac{\delta q_{t+1}}{\bar{\lambda} - \lambda_t} + \frac{\delta q_{t+1} \mu}{(1 - \mu - \lambda \mu)} = 0 \]

\[ \frac{\partial L_t}{\partial \ln k_t} = \alpha(2 + b) + \alpha \delta q_{t+1} - q_t = 0 \quad (34) \]

The second equation has a unique solution which satisfies the transversality condition and which is the constant solution:

\[ q_t = q^* \]

One obtains:

\[ q_t = q^* = \frac{\alpha(2 + b)}{1 - \alpha \delta} \quad (35) \]
Substituting this value in the first order condition is obtained a quadratic equation in terms of $\lambda^e$. After solving; we reach the optimum value for money and the optimal value of Equilibrium relationships.

**Conclusion**
Consequences of a primary section it’s possible to levied tax on money and this kind of tax could be only resource of financing the government budget and their expenditures. But infinity resource of the governments can applied it unlimitedly so in next parts in OLG models that characterized to simulated such as an economy is showed that using from this resource to financing the budget have bounds and these bounds extremely must be considered. In this model subject to characterize of each economy optimum quantity of money determined. If these bounds neglected by government then the disequilibrium will be appearance.

**REFERENCE**